

# Empirical Models for Dark Matter Halos. I. Nonparametric Construction of Density Profiles and Comparison with Parametric Models

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## ABSTRACT

We use techniques from nonparametric function estimation theory to extract the density profiles, and their derivatives, from a set of  $N$ -body dark matter halos. We consider halos generated from  $\Lambda$ CDM simulations of gravitational clustering, as well as isolated, spherical collapses. The logarithmic density slopes  $\gamma \equiv d \log \rho / d \log r$  of the  $\Lambda$ CDM halos are found to vary as power-laws in radius, reaching values of  $\gamma \approx -1$  at the innermost resolved radii,  $\sim 10^{-2} r_{\text{vir}}$ . This behavior is significantly different from that of broken power-law models like the NFW profile, but similar to that of models like de Vaucouleurs'. Accordingly, we compare the  $N$ -body density profiles with various parametric models to find which provide the best fit. We consider an NFW-like model with arbitrary inner slope; Dehnen & McLaughlin's anisotropic model; Einasto's model (identical in functional form to Sérsic's model but fit to the space density); and the density model of Prugniel & Simien that was designed to match the deprojected form of Sérsic's  $R^{1/n}$  law. Overall, the best-fitting model to the  $\Lambda$ CDM halos is Einasto's, although the Prugniel-Simien and Dehnen-McLaughlin models also perform well. With regard to the spherical collapse halos, both the Prugniel-Simien and Einasto models describe the density profiles well, with an rms scatter some four times smaller than that obtained with either the NFW-like model or the 3-parameter Dehnen-McLaughlin model. Finally, we confirm recent claims of a systematic variation in profile shape with halo mass.

*Subject headings:* dark matter — galaxies: halos — methods: N-body simulations

## 1. Introduction

A fundamental question is the distribution of matter in bound systems (galaxies, galaxy clusters, dark matter halos) that form in an expanding universe. Early work on the self-similar collapse of (spherical) primordial overdensities resulted in virialized structures having density profiles described by a single power law (e.g., Fillmore & Goldreich 1984; Bertschinger 1985; Hoffman 1988). Some of the first  $N$ -body simulations were simple cold collapse calculations like these (e.g. van Albada 1961; Aarseth 1963; Hénon 1964; Peebles 1970). It was quickly realized that, given appropriately low but non-zero levels of initial random velocity, the end state of such systems departed from a simple power law, resembling instead the de Vaucouleurs (1948)  $R^{1/4}$  profiles observed in elliptical galaxies (e.g. van Albada 1982; Aguilar & Merritt 1990). As  $N$ -body techniques improved, the logarithmic profile slopes of cold dark matter (CDM) halos, simulated in hierarchical merger models, were also observed to steepen with increasing radius (e.g., West, Dekel & Oemler 1987; Frenk et al. 1988; Efstathiou et al. 1988). Dubinski & Carlberg (1991) adopted Hernquist’s (1990) double power law model (itself a modification of Jaffe’s 1983 model) to describe these density profiles. This empirical model has an inner logarithmic slope of  $-1$  and an outer logarithmic slope of  $-4$ . It was introduced as an analytical approximation to the deprojected form of de Vaucouleurs’ (1948) profile. Navarro, Frenk, & White (1995) modified this to give the so-called NFW model that has an outer logarithmic slope of  $-3$  rather than  $-4$ , while Moore et al. (1998, 1999) suggested that a further variation having an inner logarithmic slope of  $-1.4$  or  $-1.5$  may be more appropriate.

The density profiles of  $N$ -body halos typically span only  $\sim 2$  decades in radius, between the virial radius and an inner limit set by the  $N$ -body resolution. It has long been clear that other functional forms might fit such limited data as well or better than the NFW or Moore profiles. Recently, Navarro et al. (2004) argued for a model, like de Vaucouleurs’, in which the logarithmic slope varies continuously with radius:

$$\frac{d \ln \rho}{d \ln r} = -2 \left( \frac{r}{r_{-2}} \right)^{\alpha} \quad (1)$$

i.e.

$$\rho(r) \propto \exp(-Ar^{\alpha}), \quad (2)$$

where  $r_{-2}$  is the radius at which the logarithmic slope of the density is  $-2$  and  $\alpha$  is a parameter describing the degree of curvature of the profile. Merritt et al. (2005) pointed out that this is the same relation between slope and radius that defines Sérsic’s (1963, 1968) law, with the difference that Sérsic’s law is traditionally applied to the projected (surface) densities of galaxies, not to the space density. Merritt et al. further showed that the (space) density profiles of a sample of  $N$ -body halos were equally well fit by equation (2), or by a deprojected Sérsic profile, and that both of these models provide better fits than an NFW-like, double power-law model with a variable inner slope. Hence, Sérsic’s law – the function that is so successful at describing the luminosity profiles of early-type galaxies and bulges (e.g. Caon, Capaccioli, & D’Onofrio 1993; Graham & Guzmán 2003, and references therein), and the projected density of hot gas in galaxy clusters (Demarco et al. 2003) – is also an excellent description of  $N$ -body halos.

(To limit confusion, we will henceforth refer to equation (2) as “Einasto’s  $r^{1/n}$  model” when applied to space density profiles, and as “Sérsic’s  $R^{1/n}$  model” when applied to projected density profiles, with  $R$  the radius on the plane of the sky. The former name acknowledges Einasto’s (1965, 1968, 1969) early and extensive use of equation (2) to model the light and mass distributions of galaxies (see also Einasto & Haud 1989). In addition, we henceforth replace the exponent  $\alpha$  by  $1/n$  in keeping with the usage established by Sérsic and de Vaucouleurs.)

In this paper, we continue the analysis of alternatives to the NFW and Moore profiles, using a new set of  $N$ -body halos. Among the various models that we consider is the Prugniel-Simien (1997) law, first developed as an analytic approximation to the deprojected form of the Sérsic  $R^{1/n}$  profile. Apart from the work of Lima Neto et al. (1999), Pignatelli & Galletta (1999), and Márquez et al. (2000, 2001), the Prugniel-Simien model has received little attention to date. Demarco et al. (2003) have however applied it to the gas density profiles of 24 galaxy clusters observed with ROSAT, and Terzić & Graham (2005) showed that it provides a superior description of the density profiles of real elliptical galaxies compared with

either the Jaffe or Hernquist models. As far as we are aware, ours is the first application of the Prugniel-Simien model to  $N$ -body halos.

As in Merritt et al. (2005), we base our model evaluations on *nonparametric* representations of the  $N$ -body density profiles. Such representations are “optimum” in terms of their bias-variance tradeoff, but are also notable for their flexibility. Not only do they constitute (i) “stand-alone,” smooth and continuous representations of the density and its slope: they are also well suited to (ii) inferring best-fit values for the fitting parameters of parametric functions, and (iii) comparing the goodness-of-fit of different parametric models, via the relative values of the integrated square error or a similar statistic. The more standard technique of computing binned densities is suitable (though inferior) for (ii) and (iii) but not for (i), since the density is given only at a discrete set of points and the derivatives poorly defined; while techniques like Sarazin’s (1980) maximum-likelihood algorithm provide a (perhaps) more direct route to (ii) but are not appropriate for (i) or (iii). Recently, nonparametric function estimation methods have been applied to many other problems in astrophysics, including reconstruction of the CMB fluctuation spectrum (Miller et al. 2002), dynamics of dwarf galaxies (Wang et al. 2005), and reconstruction of dark matter distributions via gravitational lensing (Abdelsalam, Saha & Williams 1998). Application of nonparametric methods to the halo density profile problem is perhaps overdue, especially given the importance of determining the inner density slope (Diemand et al. 2005).

In §2 we introduce the data sets to be analyzed. These consist of  $N$ -body simulations of ten  $\Lambda$ CDM halos and two halos formed by monolithic (nearly spherical) collapse. (Moore et al. 1999 have discussed the similarity between the end state of cold collapse simulations and hierarchical CDM models.) In §3 we present the nonparametric method used to construct the density profiles and their logarithmic slopes. §4 presents four, 3-parameter models, and §5 reports how well these empirical models perform. Our findings are summarized in Section 7.

In Paper II of this series (Graham et al. 2006a), we explore the Einasto and Prugniel-Simien models in more detail. Specifically, we explore the logarithmic slope of these models and compare

the results with observations of real galaxies. We also present the models’ circular velocity profiles and their  $\rho/\sigma^3$  profiles. Helpful expressions for the concentration and assorted scale radii:  $r_s, r_{-2}, r_e, R_e, r_{\text{vir}}$ , and  $r_{\text{max}}$  — the radius where the circular velocity profile has its maximum value — are also derived. Because the Prugniel-Simien model yields the same parameters as those coming from Sérsic-model fits, we are able to show in Paper III (Graham et al. 2006b) the location of our dark matter halos on the Kormendy diagram ( $\mu_e$  vs.  $\log R_e$ ), along with real galaxies. We additionally show in Paper III the location of our dark matter halos and real galaxies and clusters in a new  $\log(\rho_e) - \log(R_e)$  diagram.

## 2. Data: Dark matter halos

We use a sample of relaxed, dark matter halos from Diemand, Moore, & Stadel (2004a,b). Details about the simulations, convergence tests, and an estimate of the converged scales can be found in those papers. Briefly, the sample consists of six, cluster-sized halos (models: A09, B09, C09, D12, E09, and F09) resolved with 5 to 25 million particles within the virial radius, and four, galaxy-sized halos (models: G00, G01, G02, and G03) resolved with 2 to 4 million particles. The innermost resolved radii are 0.3% to 0.8% of the virial radius,  $r_{\text{vir}}$ . The outermost data point is roughly at the virial radius, which is defined in such a way that the mean density within  $r_{\text{vir}}$  is  $178\Omega_M^{0.45}\rho_{\text{crit}} = 98.4\rho_{\text{crit}}$  (Eke, Cole, & Frenk 1996) using  $\Omega_m = 0.268$  (Spergel et al. 2003). The virial radius thus encloses an overdensity which is 368 times denser than the mean matter density. We adopted the same estimates of the halo centers as in the Diemand et al. papers; these were computed using SKID (Stadel 2001), a kernel-based routine.

In an effort to study the similarities between cold, collisionless collapse halos and CDM halos, we performed two additional simulations. We distributed  $10^7$  particles with an initial density profile  $\rho(r) \propto r^{-1}$ , within a unit radius sphere with total mass 1 (M11) and 0.1 (M35). The particles have zero kinetic energy and the gravitational softening was set to 0.001. Each system collapsed and underwent a radial-orbit instability (Merritt & Aguilar 1985) which resulted in a virialized,

triaxial/prolate structure. The lower mass halo, M35, collapsed less violently over a longer period of time.

### 3. Nonparametric estimation of density profiles and their derivatives

Density profiles of  $N$ -body halos are commonly constructed by counting particles in bins. While a binned histogram is a bona-fide, non-parametric estimate of the “true” density profile, it has many undesirable properties, e.g. it is discontinuous, and it depends sensitively on the chosen size and location of the bins (see, e.g., Stepanas & Saha 1995). A better approach is to view the particle positions as a random sample drawn from some unknown, smooth density  $\rho(\mathbf{r})$ , and to use techniques from nonparametric function estimation to construct an estimate  $\hat{\rho}$  of  $\rho$  (e.g., Scott 1992). In the limit that the “sample size”  $N$  tends to infinity, such an estimate exactly reproduce the density function from which the data were drawn, as well as many properties of that function, e.g. its derivatives (Silverman 1986).

We used a kernel-based algorithm for estimating  $\rho(r)$ , similar to the algorithms described in Merritt & Tremblay (1994) and Merritt (1996). The starting point is an estimate of the 3D density obtained by replacing each particle at position  $\mathbf{r}_i$  by a kernel of width  $h_i$ , and summing the kernel densities:

$$\hat{\rho}(\mathbf{r}) = \sum_{i=1}^N \frac{m_i}{h_i^3} K \left[ \frac{1}{h_i} |\mathbf{r} - \mathbf{r}_i| \right]. \quad (3)$$

Here  $m_i$  is the mass associated with the  $i$ th particle and  $K$  is a normalized kernel function, i.e. a density function with unit volume. We adopted the Gaussian kernel,

$$K(y) = \frac{1}{(2\pi)^{3/2}} e^{-y^2/2}. \quad (4)$$

The density estimate of equation (3) has no imposed symmetries. We now suppose that  $\rho(\mathbf{r}) = \rho(r)$ , i.e. that the underlying density is spherically symmetric about the origin. In order that the density estimate have this property, we assume that each particle is smeared uniformly around the surface of the sphere whose radius is  $r_i$ . The

spherically-symmetrized density estimate is

$$\hat{\rho}(r) = \sum_{i=1}^N \frac{m_i}{h_i^3} \frac{1}{4\pi} \int d\phi \int d\theta \sin \theta K \left( \frac{d}{h_i} \right) \quad (5a)$$

$$d^2 = |\mathbf{r} - \mathbf{r}_i|^2 \quad (5b)$$

$$= r_i^2 + r^2 - 2rr_i \cos \theta \quad (5c)$$

where  $\theta$  is defined (arbitrarily) from the  $\mathbf{r}_i$ -axis. This may be expressed in terms of the angle-averaged kernel  $\tilde{K}$ ,

$$\tilde{K}(r, r_i, h_i) \equiv \frac{1}{4\pi} \int_{-\pi/2}^{\pi/2} d\phi \times \int_0^{2\pi} d\theta \sin \theta K \left( h_i^{-1} \sqrt{r_i^2 + r^2 - 2rr_i \cos \theta} \right) \quad (6a)$$

$$= \frac{1}{2} \int_{-1}^1 d\mu K \left( h_i^{-1} \sqrt{r_i^2 + r^2 - 2rr_i \mu} \right), \quad (6b)$$

as

$$\hat{\rho}(r) = \sum_{i=1}^N \frac{m_i}{h_i^3} \tilde{K}(r, r_i, h_i). \quad (7)$$

Substituting for the Gaussian kernel, we find

$$\tilde{K}(r, r_i, h_i) = \frac{1}{(2\pi)^{3/2}} \left( \frac{rr_i}{h_i^2} \right)^{-1} \times \exp \left[ -(r_i^2 + r^2)/2h_i^2 \right] \sinh(rr_i/h_i^2). \quad (8)$$

A computationally preferable form is

$$\tilde{K} = \frac{1}{2(2\pi)^{3/2}} \left( \frac{rr_i}{h_i^2} \right)^{-1} \times \left\{ \exp \left[ -(r_i - r)^2/2h_i^2 \right] - \exp \left[ -(r_i + r)^2/2h_i^2 \right] \right\} \quad (9)$$

Equations (7) and (9) define the density estimate. Typically, one sets up a grid in radius and evaluates  $\hat{\rho}(r)$  discretely on the grid. However we stress that the density estimate itself is a continuous function and is defined independently of any grid.

Given a sample of  $N$  positions and particle masses drawn randomly from some (unknown)  $\rho(r)$ , the goal is to construct an estimate  $\hat{\rho}(r)$  that is as close as possible, in some sense, to  $\rho(r)$ . In the scheme just described, one has the freedom to adjust the  $N$  kernel widths  $h_i$  in order to achieve this. In general, if the  $h_i$  are too small, the density estimate will be “noisy,” i.e.  $\hat{\rho}(r)$  will exhibit a large *variance* with respect to the true density; while

if the  $h_i$  are too large, the density estimate will be over-smoothed, i.e. there will be a large *bias*. (Of course the same is true for binned histograms, although in general the bias-variance tradeoff for histograms is less good than for kernel estimates.) If the true  $\rho(r)$  were known *a priori*, one could adjust the  $h_i$  so as to minimize (say) the mean square deviation between  $\rho(r)$  and  $\hat{\rho}(r)$ . Since  $\rho(r)$  is not known *a priori* for our halos, some algorithm must be adopted for choosing the  $h_i$ . We followed the standard practice (e.g. Silverman 1986, p.101) of varying the  $h_i$  as a power of the local density:

$$h_i = h_0 [\hat{\rho}_{\text{pilot}}(r_i)/g]^{-\alpha}, \quad (10)$$

where  $\hat{\rho}_{\text{pilot}}(r)$  is a “pilot” estimate of  $\rho(r)$ , and  $g$  is the geometric mean of the pilot densities at the  $r_i$ . Since the pilot estimate is used only for assigning the  $h_i$ , it need not be differentiable, and we constructed it using a nearest-neighbor scheme.

The final density estimate  $\hat{\rho}(r)$  is then a function of two quantities:  $h_0$  and  $\alpha$ . Figure 1 illustrates the dependence of  $\hat{\rho}(r)$  on  $h_0$  when the kernel algorithm is applied to a random sample of  $10^6$  equal-mass particles generated from an Einasto density profile with  $n = 5$ , corresponding to typical values observed in Merritt et al. (2005). Each of the density profile estimates of Figure 1 used  $\alpha = 0.3$ . As expected, for small  $h_0$ , the estimate of  $\rho(r)$  is noisy, but faithful in an average way to the true profile; while for large  $h_0$ ,  $\rho(r)$  is a smooth function but is biased at small radii due to the averaging effect of the kernel. For  $\alpha = 0.3$ , The “optimum”  $h_0$  for this sample is  $\sim 0.05 r_e$ , where  $r_e$  is the half-mass radius coming from the Einasto model (see Section 4.2).

In what follows, we will compare the nonparametric estimates  $\hat{\rho}(r)$  derived from the  $N$ -body models with various parametric fitting functions, in order to find the best-fitting parameters of the latter by minimizing the rms residuals between the two profiles. For this purpose, any of the density estimates in Figure 1 would yield similar results, excepting perhaps the density estimate in the uppermost panel which is clearly biased at small radii. In addition, we will also wish to characterize the rms value of the deviation between the “true” profile and the best-fitting parametric models. Here it is useful for the kernel widths to be chosen such that the residuals are dominated by the systematic differences between the parametric

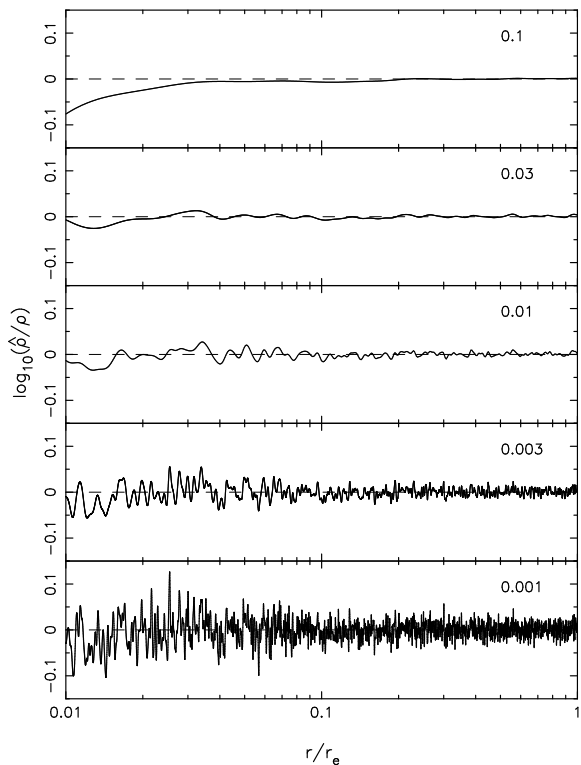


Fig. 1.— Nonparametric, bias-variance trade-off in the estimation of  $\rho(r)$  using a single sample of  $10^6$  radii generated from a halo having an Einasto  $r^{1/n}$  density profile with  $n = 5$  (see Section 4.2). From top to bottom,  $h_0 = (0.1, 0.03, 0.01, 0.003, 0.001) r_e$ ; all estimates used  $\alpha = 0.3$  (see equations 7, 9, and 10).

and nonparametric profiles, and not by noise in  $\hat{\rho}(r)$  resulting from overly-small kernels. We verified that this condition was easily satisfied for all of the  $N$ -body models analyzed here: there was always found to be a wide range of  $(h_0, \alpha)$  values such that the residuals between  $\hat{\rho}(r)$  and the parametric function were nearly constant with varying radial coordinate. This is a consequence of the large particle numbers ( $> 10^6$ ) in the  $N$ -body models, which imply a low variance even for small  $h_0$ .

As discussed above, quantities like the derivative of the density can also be computed directly from  $\hat{\rho}(r)$ . Figure 2 shows nonparametric estimates of the slope,  $d \log \rho / d \log r$ , for the same  $10^6$  particle data set as in Figure 1. We computed derivatives simply by numerically differentiating  $\hat{\rho}(r)$ ; alternatively, we could have differentiated equation (9). Figure 2 shows that as  $h_0$  is increased, the variance in the estimated slope drops, and for  $h_0 \approx 0.2 r_e$  the estimate is very close to the true function. We note that the optimal choice of  $h_0$  when estimating derivatives is larger than when estimating  $\rho(r)$  ( $\sim 0.2 r_e$  vs.  $\sim 0.05 r_e$ ); this is a well-known consequence of the increase in “noise” associated with differentiation. Figure 2 also illustrates the important point that there is no need to impose an additional level of smoothing when computing the density derivatives (as was done, e.g., in Reed et al. 2005); it is sufficient to increase  $h_0$ .

### 3.1. Application to the $N$ -body halos

Figure 3 shows, using  $\alpha = 0.3$  and  $h_0 = 0.05 r_e$  (left panel) and  $\alpha = 0.4$  and  $h_0 = 0.05 r_e$  (right panel)<sup>1</sup>, the nonparametric estimates of  $\rho(r)$  (left panel) and  $\gamma(r) \equiv d \log \rho / d \log r$  (right panel) for the ten  $N$ -body halos. Figure 4 shows the same quantities for the two data sets generated from cold collapses. We stress that these plots – especially, the derivative plots – could not have been made from tables of binned particle numbers. For most profiles, the slope is a rather continuous function of radius and does not appear to reach any obvious, asymptotic, central value by  $\sim 0.01 r_{\text{vir}}$ . Instead,  $\hat{\gamma}(r)$  varies approximately as a power of  $r$ , i.e.  $\log \hat{\gamma}$  vs.  $\log r$  is approximately a straight

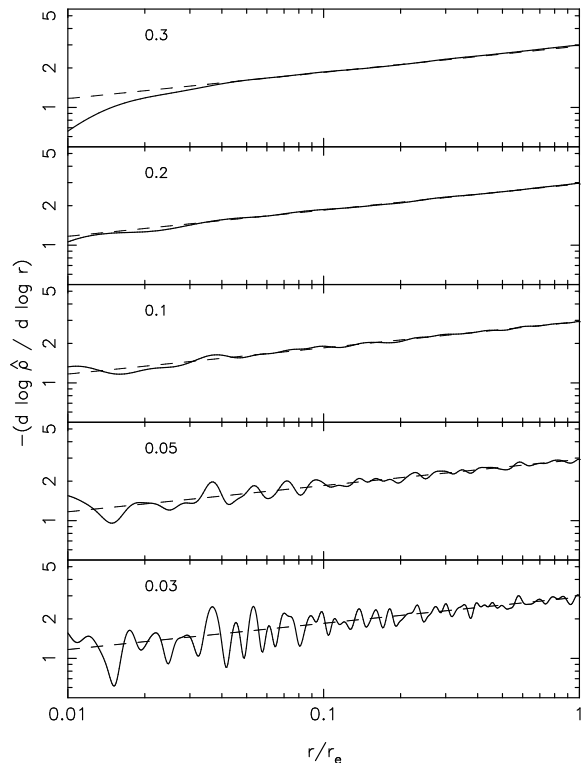


Fig. 2.— Five estimates of the logarithmic slope of an Einasto  $r^{1/n}$  halo, derived via differentiation of  $\hat{\rho}(r)$ . The same sample of  $10^6$  radii was used as in Figure 1. From top to bottom,  $h_0 = (0.3, 0.2, 0.1, 0.05, 0.03) r_e$ ; each estimate used  $\alpha = 0.4$  (see equation 10). Dashed lines show the true slope.

<sup>1</sup>We have purposely used a small value of  $h_0$  to avoid any possibility of biasing the slope estimates.

line. Accordingly, we have fitted straight lines, via a least-squares minimization, to the logarithmic profile slopes in the right-hand panels of Figures 3 and 4. The regression coefficients, i.e. slopes, are inset in each panel. (These slope estimates should be seen as indicative only; they are superseded by the model fits discussed below.) In passing we note that such a power-law dependence of  $\gamma$  on  $r$  is characteristic of the Einasto model, with the logarithmic slope equal to the exponent  $1/n$ . Noise and probable (small) deviations from a perfect Einasto  $r^{1/n}$  model are expected to produce slightly different exponents when we fit the density profiles in the following Section with Einasto's  $r^{1/n}$  model and a number of other empirical functions.

The slope at the innermost resolved radius is always close to  $-1$ , which is also the slope at  $r = 0$  in the NFW model. However there is no indication in Figure 3 that  $\hat{\gamma}(r)$  is flattening at small radii, i.e., it is natural to conclude that  $N$ -body simulations of higher resolution would exhibit smaller inner slopes. On average, the slope at  $r_{\text{vir}}$  is around  $-3$ , but there are large fluctuations and some halos reach a value of  $-4$ , as previously noted in Diemand et al. (2004b). The reason for these fluctuation may be because the outer parts are dynamically very young (i.e. measured in local dynamical times) and they have only partially completed the violent relaxation into a stable, stationary equilibrium configuration. We are not able to say with any confidence what the slopes do beyond  $r_{\text{vir}}$ .

#### 4. Empirical models

In this section, we present four parametric density models, each having three independent parameters: two “scaling” parameters and one “shape” parameter. We measured the quality of each parametric model's fit to the nonparametric  $\hat{\rho}(r)$ 's using a standard metric, the integrated square deviation,

$$\int d(\log r) [\log \hat{\rho}(r) - \log \rho_{\text{param}}(r)]^2 \quad (11)$$

where  $\rho_{\text{param}}$  is understood to depend on the various fitting parameters as well as on  $r$ . Equation (11) is identical in form to the Cramér-von Mises statistic (e.g. Cox & Hinkley 1974, eq. 6), an alternative to the Kolmogorov-Smirnov statis-

tic for comparing two (cumulative) distribution functions.

We chose to evaluate this integral by discrete summation on a grid spaced uniformly in  $\log r$ ; our measure of goodness-of-fit (which was also the quantity that was minimized in determining the best-fit parameters) was

$$\Delta^2 \equiv \frac{\sum_{j=1}^m \delta_j^2}{m-3}, \quad (12a)$$

$$\delta_j = \log_{10}[\hat{\rho}(r_j)/\rho_{\text{param}}(r_j)] \quad (12b)$$

with  $m = 300$ . With such a large value of  $m$  the results obtained by minimizing (12a) and (11) are indistinguishable. We note that the quantity  $\Delta^2$  in equation (12a) is reminiscent of the standard  $\chi^2$ , but the resemblance is superficial. For instance,  $\Delta^2$  as defined here is independent of  $m$  in the large- $m$  limit (and our choice of  $m = 300$  puts us effectively in this limit). Furthermore there is no binning involved in the computation of  $\Delta^2$ ; the grid is simply a numerical device used in the computation of (11).

##### 4.1. Double power-law models

Hernquist (1990, his equation 43) presented a 5-parameter generalization of Jaffe's (1983) double power-law model. Sometimes referred to as the  $(\alpha, \beta, \gamma)$  model, it can be written as

$$\rho(r) = \rho_s 2^{(\beta-\gamma)/\alpha} \left(\frac{r}{r_s}\right)^{-\gamma} \left[1 + \left(\frac{r}{r_s}\right)^\alpha\right]^{(\gamma-\beta)/\alpha}, \quad (13)$$

where  $\rho_s$  is the density at the scale radius,  $r_s$ , which marks the center of the transition region between the inner and outer power-laws having slopes of  $-\gamma$  and  $-\beta$ , respectively. The parameter  $\alpha$  controls the sharpness of the transition (see Zhao et al. 1996; Kravtsov et al. 1998; and equations (37) and (40b) in Dehnen & McLaughlin 2005). Setting  $(\alpha, \beta, \gamma) = (1, 3, 1)$  yields the NFW model, while  $(1.5, 3, 1.5)$  gives the model in Moore et al. (1999). Other combinations have been used, for example,  $(1, 3, 1.5)$  was applied in Jing & Suto (2000) and  $(1, 2.5, 1)$  was used by Rasia, Tormen, & Moscardini (2004).

In fitting dark matter halos, Klypin et al. (2001, their figure 8) have noted a certain degree of degeneracy when all of the 5 parameters are allowed to vary. Graham et al. (2003, their figures 3 and

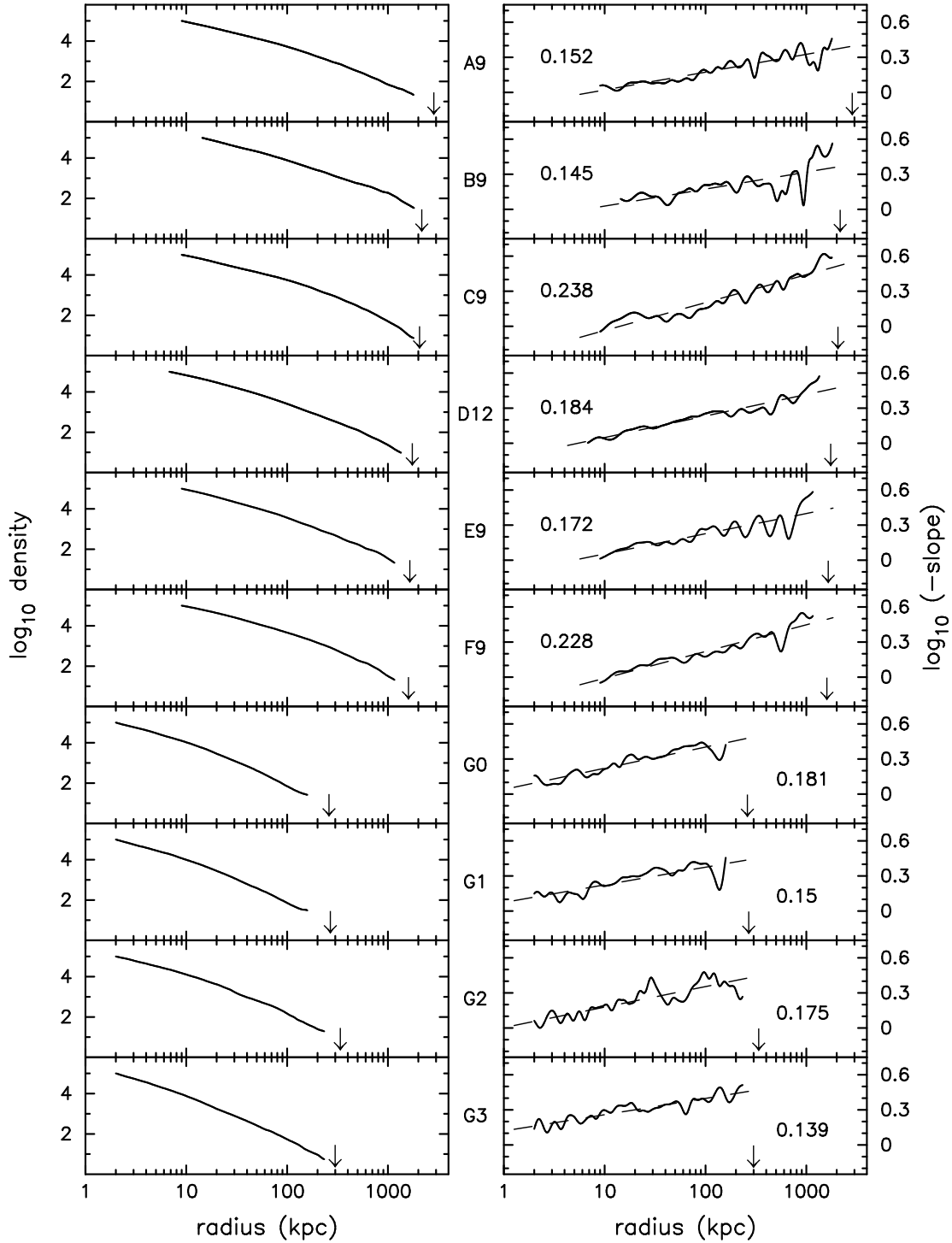


Fig. 3.— Nonparametric estimates of the density  $\rho(r)$  (left panel) and the slope  $d \log \rho / d \log r$  (right panel) for the ten  $N$ -body halos of Table 1. The virial radius  $r_{vir}$  is marked with an arrow. Dashed lines in the right hand panels are linear fits of  $\log(-d \log \rho / d \log r)$  to  $\log r$ ; regression coefficients are also given.



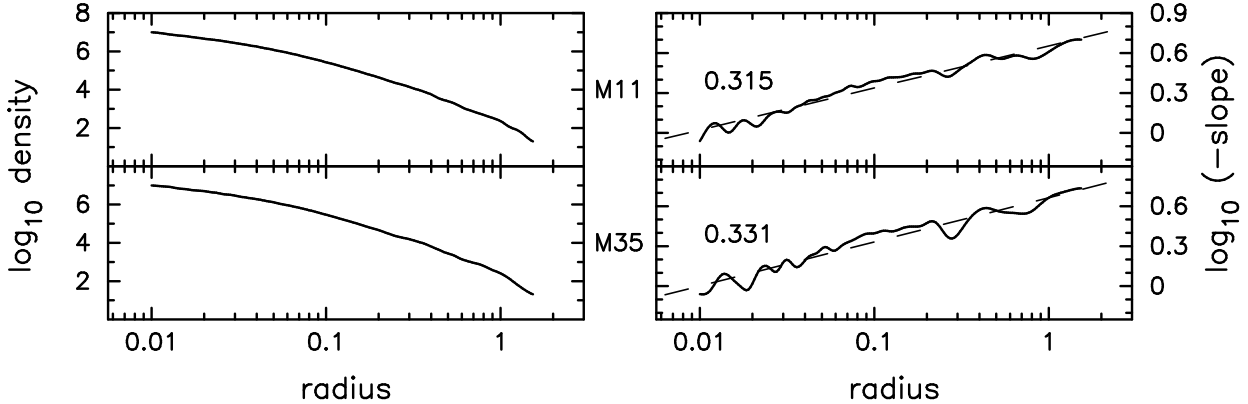


Fig. 4.— Nonparametric estimates of  $\rho(r)$  (left panel) and  $d \log \rho / d \log r$  (right panel) for the two “collapse” models. Dashed lines in the right hand panels are linear fits of  $\log(-d \log \rho / d \log r)$  to  $\log r$ .

4) have also observed the parameters of this empirical model to be highly unstable when applied to (light) profiles having a continuously changing logarithmic slope. Under such circumstances, the parameters can be a strong function of the fitted radial extent, rather than reflecting the intrinsic physical properties of the profile under study. This was found to be the case when applied to the dark matter halos under study here. We have therefore chosen to constrain two of the model parameters, holding  $\alpha$  fixed at 1 and  $\beta$  fixed at 3.

In recent years, as the resolution in  $N$ -body simulations has improved, Moore and collaborators have found that the innermost (resolved) logarithmic slope of dark matter halos has a range of values which are typically shallower than  $-1.5$ : recently obtaining a mean value ( $\pm$  standard deviation) equal to  $-1.26 \pm 0.17$  at 1% of the virial radius (Diemand, Moore, & Stadel 2004b). At the same time, Navarro et al. (2004) report that the NFW model underestimates the density over the inner regions of most of their halos, which have innermost resolved slopes ranging from  $-1.6$  to  $-0.95$  (their Figure 3). A model with an outer slope of  $-3$  and an inner slope of  $-\gamma$  might therefore be more appropriate. Such a model has been used before and can be written as

$$\rho(r) = \frac{2^{3-\gamma} \rho_s}{(r/r_s)^\gamma (1 + r/r_s)^{3-\gamma}}. \quad (14)$$

The total mass of this model is infinite however.

We have applied the above  $(1, 3, \gamma)$  model to our dark matter density profiles, the results of

which are shown in Figure 5 for the  $N$ -body halos, and in the upper panel of Figure 6 for the cold collapse models. The rms scatter,  $\Delta$ , is inset in each figure and additionally reported in Table 1.

#### 4.1.1. Two-parameter models

Recognizing that galaxies appear to have flat inner density profiles (e.g., Flores & Primack 1994; Moore 1994), Burkert (1995) cleverly introduced a density model having an inner slope of zero and an outer profile that decayed as  $r^{-3}$ . His model is given by the expression

$$\rho(r) = \frac{\rho_0 r_s^3}{(r + r_s)(r^2 + r_s^2)}, \quad (15)$$

where  $\rho_0$  is the central density and  $r_s$  is a scale radius. Application of this model in Figure 7 reveals that, with only 2 free parameters, it does not provide as good a fit to the simulated dark matter halos as the  $(1, 3, \gamma)$  model presented above. The hump-shaped residual profiles in Figure 7 signify the model’s inability to match the curvature of our density profiles. (It is important to point out that Burkert’s model was introduced to fit the observed rotation curves in low surface brightness galaxies, after the contribution from the baryonic component had been subtracted out, a task which it performs well.)

As noted previously, the NFW  $(\alpha, \beta, \gamma) = (1, 3, 1)$  model also has only two parameters:  $\rho_s$  and  $r_s$ . Because this model is still often used, we apply it to our halos in Figure 8. Comparison with Figure 5

TABLE 1  
THREE-PARAMETER MODELS

Halo Id.	$r_s$ kpc	$\log \rho_s$ $M_\odot \text{ pc}^{-3}$	$\gamma$	$\Delta$ dex	$r_e$ kpc	$\log \rho_e$ $M_\odot \text{ pc}^{-3}$	$n_{\text{Ein}}$	$\Delta$ dex	$R_e$ kpc	$\log \rho'$ $M_\odot \text{ pc}^{-3}$	$n_{\text{PS}}$	$\Delta$ dex
(1,3, $\gamma$ )					Einasto $r^{1/n}$				Prugniel-Simien			
Cluster-sized halos												
A09	626.9	−3.87	1.174	0.025	5962.	−6.29	6.007	<b>0.015</b>	2329.	−2.73	3.015	0.021
B09	1164.	−4.75	1.304	<b>0.037</b>	17380.	−7.66	7.394	0.041	4730.	−3.34	3.473	0.038
C09	241.8	−3.27	0.896	0.040	1247.	−4.95	3.870	0.030	738.9	−2.55	2.192	<b>0.016</b>
D12	356.1	−3.82	1.251	0.026	2663.	−6.02	5.939	0.020	1232.	−2.52	3.147	<b>0.019</b>
E09	382.5	−3.96	1.265	0.033	2611.	−6.06	5.801	0.032	1231.	−2.62	3.096	<b>0.030</b>
F09	233.9	−3.51	1.012	0.030	1235.	−5.26	4.280	0.025	697.3	−2.63	2.400	<b>0.017</b>
Galaxy-sized halos												
G00	27.96	−3.16	1.163	<b>0.020</b>	189.0	−5.22	5.284	0.023	114.4	−2.02	3.135	0.028
G01	35.34	−3.36	1.275	0.029	252.6	−5.51	5.873	<b>0.028</b>	146.0	−2.01	3.425	0.032
G02	53.82	−3.59	1.229	0.034	391.4	−5.74	5.725	<b>0.031</b>	214.9	−2.34	3.243	0.036
G03	54.11	−3.70	1.593	0.028	405.6	−5.98	7.791	<b>0.023</b>	229.1	−1.47	4.551	0.024
Spherical collapse halos												
M11	0.0175	2.66	0.006	0.223	0.244	0.27	3.426	<b>0.043</b>	0.187	2.57	2.445	0.051
M35	0.0180	1.62	0.030	0.249	0.240	−0.70	3.214	<b>0.059</b>	0.185	1.47	2.301	0.061

NOTE.—Col.(1): Object Id. Col.(2)–(5) (1, 3,  $\gamma$ ) model (equation 13 and 14) scale radius  $r_s$ , scale density  $\rho_s$ , inner profile slope  $\gamma$ , and rms scatter of the fit. Col.(6)–(9) Einasto  $r^{1/n}$  model half-mass radius  $r_e$ , associated density  $\rho_e$ , profile shape  $n_{\text{Ein}}$ , and rms scatter of the fit. Col.(10)–(13) Prugniel-Simien model scale radius  $R_e$ , scale density  $\rho'$  (the spatial density  $\rho_e$  at  $r = R_e$  is such that  $\rho_e = \rho' e^{-b}$ ), profile shape  $n_{\text{PS}}$ , and rms scatter of the fit. Note: The radius and density units do not apply to M11 and M35. For each halo, of the three models shown here the model having the lowest residual scatter is high-lighted in bold.

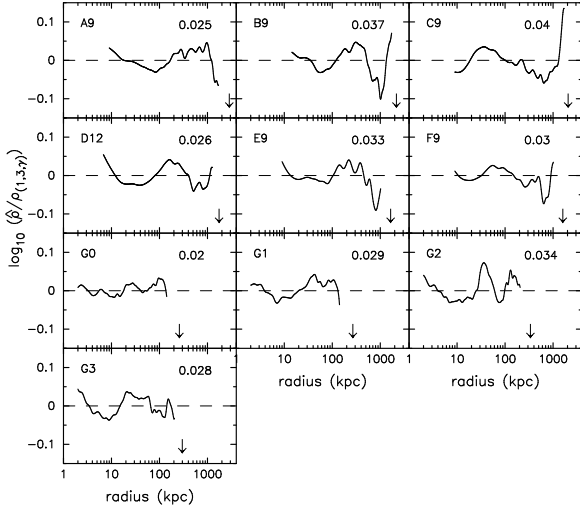


Fig. 5.— Residual profiles from application of the 3-parameter (1, 3,  $\gamma$ ) model (equation 14) to our ten,  $N$ -body density profiles. The virial radius is marked with an arrow, and the rms residual (equation 12a) is inset with the residual profiles.

reveals that the NFW model never performs better than the (1, 3,  $\gamma$ ) model; the residuals are  $\sim 50\%$  larger and sometimes twice as large. Importantly, the large-scale curvature observed in many of the NFW residual profiles (Figure 8) reveals that this model does not describe the majority of the halos, and that the (1, 3,  $\gamma$ ) model should be preferred over the NFW model.

An alternative 2-parameter expression has recently been studied by Dehnen & McLaughlin (2005, their equation (20b); see also Austin et al. 2005). It is a special case of a more general family of models — which we test next — when the velocity ellipsoid at the halo center is isotropic and  $\rho/\sigma_r^3$  is a (special) power law in radius, varying as  $r^{-35/18}$ . This 2-parameter density model is an  $(\alpha, \beta, \gamma) = (4/9, 31/9, 7/9)$  model given by

$$\rho(r) = \frac{2^6 \rho_s}{(r/r_s)^{7/9} [1 + (r/r_s)^{4/9}]^6}, \quad (16)$$

and is applied in Figure 9. It clearly provides a much better match to the dark matter halo density profiles in comparison with the previous 2-parameter model over the fitted radial range, but

TABLE 2  
THREE-PARAMETER MODELS (*cont.*)

Halo Id.	$r_s$ kpc	$\log \rho_s$ $M_\odot \text{ pc}^{-3}$	$\gamma'$	$\Delta$ dex
Anisotropic Dehnen-McLaughlin (Eq.17)				
Cluster-sized halos				
A09	722.7	−2.21	0.694	<b>0.013</b>
B09	1722.	−3.30	0.880	0.040
C09	207.0	−1.34	0.241	0.047
D12	322.8	−1.95	0.683	0.022
E09	330.4	−2.04	0.669	0.034
F09	193.6	−1.56	0.350	0.036
Galaxy-sized halos				
G00	20.89	−1.11	0.422	<b>0.017</b>
G01	25.88	−1.28	0.568	<b>0.023</b>
G02	43.05	−1.60	0.581	<b>0.027</b>
G03	30.20	−1.34	0.849	0.024
Spherical collapse halos				
M11	0.025	4.23	0.00	0.179
M35	0.025	3.21	0.00	0.206

NOTE.—Col.(1): Object Id. Col.(2)–(5) Dehnen-McLaughlin (their equation 46b) scale radius  $r_s$ , scale density  $\rho_s$ , inner profile slope  $\gamma'$ , and rms scatter of the fit. Note: The radius and density units do not apply to M11 and M35. When the rms scatter is lower than the value obtained with the other 3-parameter models, it is given in bold.

TABLE 3  
TWO-PARAMETER MODELS

Halo Id.	$r_s$ kpc	$\log \rho_0$ $M_\odot \text{ pc}^{-3}$	$\Delta$ dex	$r_s$ kpc	$\log \rho_s$ $M_\odot \text{ pc}^{-3}$	$\Delta$ dex	$r_s$ kpc	$\log \rho_s$ $M_\odot \text{ pc}^{-3}$	$\Delta$ dex
	Burkert				NFW		Isotropic Dehnen-McLaughlin (equation 16)		
					Cluster-sized halos				
A09	114.0	−1.65	0.242	419.8	−3.50	0.042	933.7	−2.43	<b>0.018</b>
B09	145.2	−2.23	0.247	527.2	−4.03	0.068	1180.0	−2.97	<b>0.042</b>
C09	96.16	−1.74	0.181	284.4	−3.42	<b>0.042</b>	554.3	−2.27	0.091
D12	68.39	−1.62	0.230	213.3	−3.34	0.051	409.1	−2.17	<b>0.026</b>
E09	77.09	−1.80	0.215	227.0	−3.46	0.053	428.2	−2.28	<b>0.037</b>
F09	80.17	−1.85	0.181	229.0	−3.49	<b>0.030</b>	438.2	−2.32	0.066
					Galaxy-sized halos				
G00	10.12	−1.56	0.139	22.23	−2.94	<b>0.024</b>	34.43	−1.59	0.037
G01	10.28	−1.54	0.152	23.12	−2.95	0.038	36.53	−1.61	<b>0.031</b>
G02	14.06	−1.66	0.183	36.39	−3.22	0.044	63.06	−1.96	<b>0.035</b>
G03	09.35	−1.32	0.179	19.54	−2.68	0.066	26.98	−1.23	<b>0.025</b>
					Spherical collapse halos				
M11	0.0261	3.01	<b>0.203</b>	0.0309	2.23	0.233	0.0234	4.31	0.244
M35	0.0265	1.98	<b>0.231</b>	0.0314	1.20	0.259	0.0236	3.29	0.269

NOTE.—Col.(1): Object Id. Col.(2)–(4) Burkert (1995) model scale radius  $r_s$ , central density  $\rho_0$ , and rms scatter of the fit (using  $m - 2$  in the denominator of equation refEqChi). Col.(5)–(7) NFW (1, 3, 1) model scale radius  $r_s$ , scale density  $\rho_0$ , and rms scatter of the fit (using  $m - 2$ ). Col.(8)–(10) Dehnen-McLaughlin (2005, their equation 20b) model scale radius  $r_s$ , associated density  $\rho_s$ , and rms scatter (using  $m - 2$ ). This model has an inner and outer, negative logarithmic slope of  $7/9 \approx 0.78$  and  $31/9 \approx 3.44$ , respectively. Note: The above radius and density units do not apply to M11 and M35. For each halo, the 2-parameter model with the lowest residual scatter is high-lighted in bold.

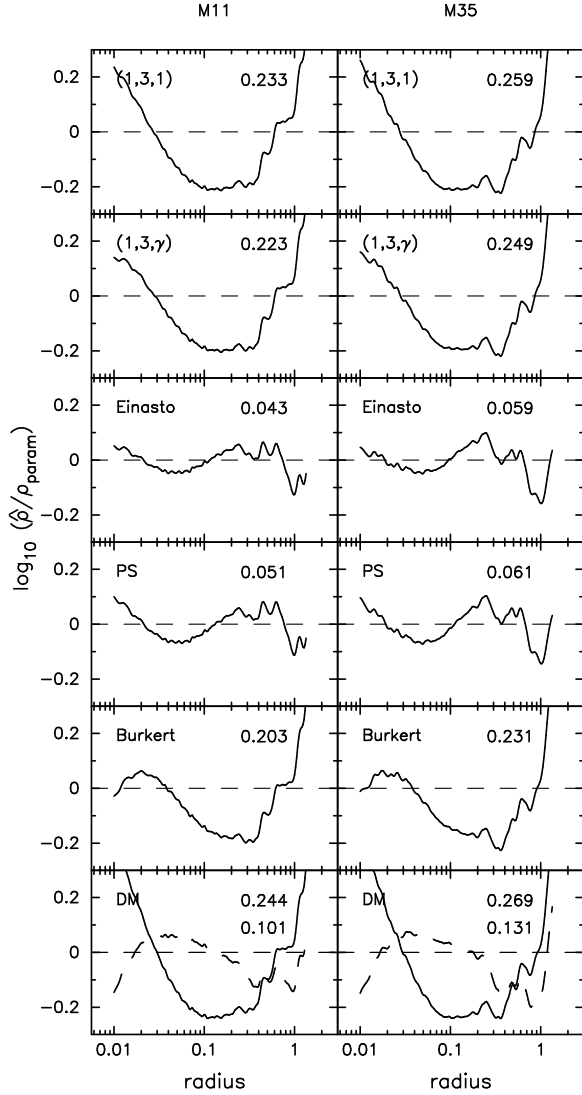


Fig. 6.— Residual profiles from the application of seven different parametric models (see Section 4) to our “cold collapse” density halos, M11 and M35. Einasto’s model, which has the same functional form as Sérsic’s model, is labelled ‘Sérsic’ in this Figure. In the lower panel, the solid curve corresponds to the 2-parameter model from Dehnen & McLaughlin (2005), and the dashed curve corresponds to their 3-parameter model. The rms residual (equation 12a) is inset in each figure.

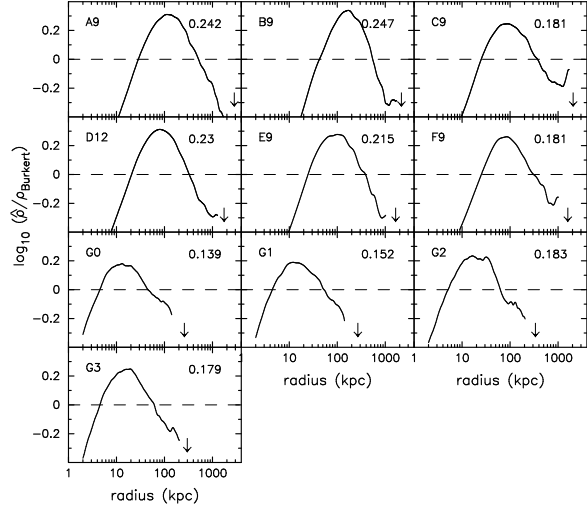


Fig. 7.— Residual profiles from application of Burkert’s 2-parameter model (equation 15) to our dark matter density profiles.

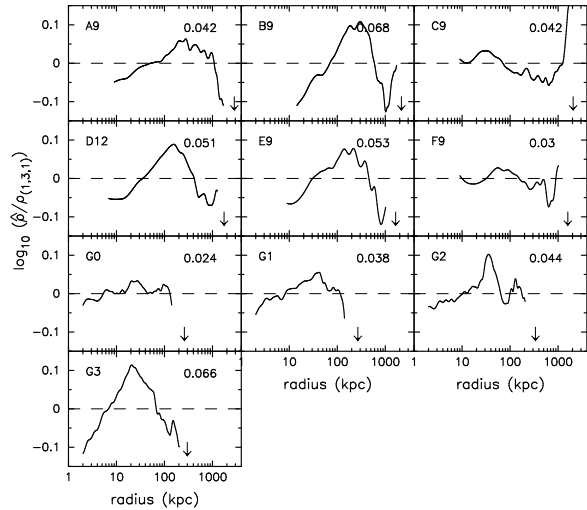


Fig. 8.— Residual profiles from application of the 2-parameter NFW (1, 3, 1) model to our dark matter density profiles.

the rms scatter reveals that it does not perform as well as the  $(1, 3, \gamma)$  model, nor can it describe the ‘spherical collapse’ halos (Figure 6). We therefore, in the following subsection, test the more general 3-parameter model given in Dehnen & McLaughlin (2005).

#### 4.1.2. Dehnen-McLaughlin’s anisotropic 3-parameter model

Dehnen & McLaughlin (2005, their equation 46b) present a theoretically-motivated, 3-parameter model such that  $[\alpha, \beta, \gamma] = [2(2 - \beta_0)/9, (31 - 2\beta_0)/9, (7 + 10\beta_0)/9]$ , and the term  $\beta_0$  reflects the central ( $r = 0$ ) anisotropy — a measure of the tangential to radial velocity dispersion<sup>2</sup>. Setting  $\gamma' = (7 + 10\beta_0)/9$ , we have  $[\alpha, \beta, \gamma] = [(3 - \gamma')/5, (18 - \gamma')/5, \gamma']$ , and their density model can be written as

$$\rho(r) = \frac{2^6 \rho_s}{(r/r_s)^{\gamma'} [1 + (r/r_s)^{(3-\gamma')/5}]^6}, \quad (17)$$

As shown in Figure 10, for three of the six cluster-sized halos, this model has the greatest residual scatter of the four, 3-parameter models tested here. For another two of the six cluster-sized halos it has the second greatest residual scatter. This model is also unable to match the curvature in the halos of the cold collapse models (Figure 10). However, it does provide very good fits to the galaxy-sized halos, and actually has the smallest residual scatter for three of these halos (Table 2).

The shallowest, inner, negative logarithmic slope of this model occurs when  $\beta_0 = 0$ , giving a value of  $7/9 \approx 0.78$ . For non-zero values of  $\beta_0$ , this slope steepens roughly linearly with  $\beta_0$ .

#### 4.2. Sérsic/Einasto model

Sérsic (1963, 1968) generalized de Vaucouleurs’ (1948)  $R^{1/4}$  luminosity profile model by replacing the exponent  $1/4$  with  $1/n$ , such that  $n$  was a free parameter that measured the ‘shape’ of a galaxy’s luminosity profile. Using the observers’ notion of ‘concentration’ (see the review in Graham, Trujillo, & Caon 2001), the quantity  $n$  is

<sup>2</sup>Note: the quantities  $\beta$  and  $\beta_0$  are not as related as their notation suggests. The former is the outermost, negative logarithmic slope of the density profile while the latter is the velocity anisotropy parameter at  $r = 0$ .

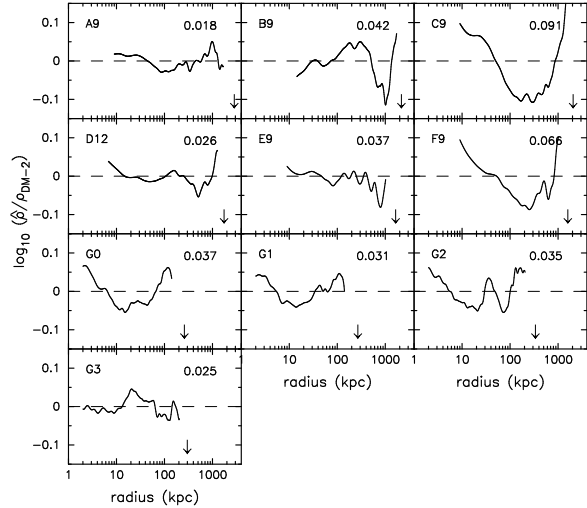


Fig. 9.— Residual profiles from application of the 2-parameter  $(4/9, 31/9, 7/9)$  model (equation 16) from Dehnen & McLaughlin (2005, their equation 20b) to our dark matter density profiles.

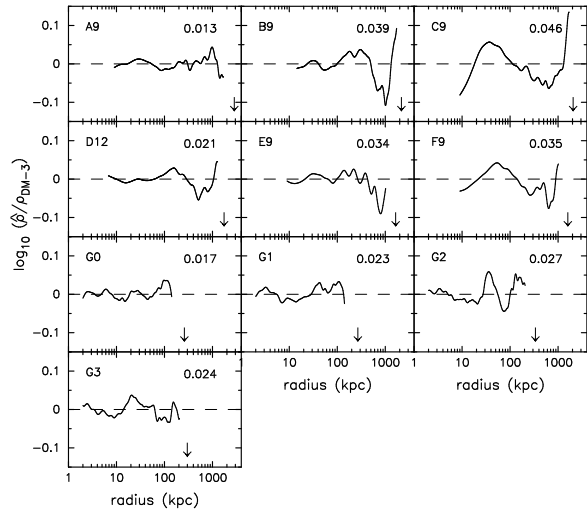


Fig. 10.— Residual profiles from application of the 3-parameter  $[(3 - \gamma')/5, (18 - \gamma')/5, \gamma']$  model (equation 17) from Dehnen & McLaughlin (2005, their equation 46b) to our dark matter density profiles.

monotonically related to how centrally concentrated a galaxy's light profile is. With  $R$  denoting the *projected* radius, Sérsic's  $R^{1/n}$  model is often written as

$$I(R) = I_e \exp \left\{ -b_n \left[ (R/R_e)^{1/n} - 1 \right] \right\}, \quad (18)$$

where  $I_e$  is the (projected) intensity at the (projected) effective radius  $R_e$ . The term  $b_n$  is not a parameter but a function of  $n$  and defined in such a way that  $R_e$  encloses half of the (projected) total galaxy light (Caon et al. 1993; see also Ciotti 1991, his equation (1)). A good approximation when  $n \gtrsim 0.5$  is given in Prugniel & Simien (1997) as

$$b_n \approx 2n - 1/3 + 0.009876/n. \quad (19)$$

Assorted expressions related to the  $R^{1/n}$  model can be found in Graham & Driver's (2005) review article.

Despite the success of this model in describing the luminosity profiles of elliptical galaxies (e.g., Phillipps et al. 1998; Caon et al. 1993; D'Onofrio et al. 1994; Young & Currie 1995; Graham et al. 1996; Graham & Guzmán 2003, and references therein), it is nonetheless an empirical fitting function with no commonly recognized physical basis. We are therefore free to explore the suitability of this function for describing the *mass* density profiles,  $\rho(r)$ , of dark matter halos. Indeed, Einasto (1965, eq. 4; 1968, eq. 1.7; 1969, eq. 3.1) independently developed the functional form of Sérsic's equation and used it to describe density profiles. More recent application of this profile to the modelling of density profiles can be found in Einasto & Haud (1989, their eq. 14) and Tenjes, Haud, & Einasto (1994, their eq. A1). Most recently, the same model has been applied by Navarro et al. (2004) and Merritt et al. (2005) to characterize dark matter halos, and Aceves, Velázquez, & Cruz's (2006) used it to describe merger remnants in simulated disk galaxy collisions.

To avoid potential confusion with Sérsic's  $R^{1/n}$  model, we define the following expression as "Einasto's  $r^{1/n}$  model":

$$\rho(r) = \rho_e \exp \left\{ -d_n \left[ (r/r_e)^{1/n} - 1 \right] \right\}, \quad (20)$$

where  $r$  is the *spatial* (i.e., not projected) radius. The term  $d_n$ , defined below, is a function of  $n$  such that  $\rho_e$  is the density at the radius  $r_e$  which defines

a volume containing half of the total mass. The central density is finite and given by  $\rho(r=0) = \rho_e e^{d_n}$ .

The integral of equation (20) over some volume gives the enclosed mass<sup>3</sup>, which is also finite and equal to

$$M(r) = 4\pi \int_0^r \rho(\bar{r}) \bar{r}^2 d\bar{r}. \quad (21)$$

This can be solved by using the substitution  $\bar{x} \equiv d_n(\bar{r}/r_e)^{1/n}$  to give

$$M(r) = 4\pi n r_e^3 \rho_e e^{d_n} d_n^{-3n} \gamma(3n, x), \quad (22)$$

where  $\gamma(3n, x)$  is the incomplete gamma function defined by

$$\gamma(3n, x) = \int_0^x e^{-t} t^{3n-1} dt. \quad (23)$$

Replacing  $\gamma(3n, x)$  with  $\Gamma(3n)$  in equation (22) gives the total mass  $M_{tot}$ .

The value of  $d_n$ , which we first saw in equation (20), is obtained by solving  $\Gamma(3n) = 2 \times \gamma(3n, d_n)$ , where  $\Gamma$  is the (complete) gamma function. The value of  $d_n$  can be well approximated (Mamon 2005, priv. comm.) by the expression

$$d_n \approx 3n - 1/3 + 0.0079/n, \text{ for } n \gtrsim 0.5 \quad (24)$$

(see Figure 11).

In Paper II we recast Einasto's  $r^{1/n}$  model using the radius  $r_{-2}$ , where the logarithmic slope of the density profile equals  $-2$ .

Einasto's  $r^{1/n}$  model (see Einasto & Haud 1989) was used in Navarro et al. (2004; their equation 5) to fit their simulated dark matter halos. They obtained  $n \approx 1/(0.172 \pm 0.032) \approx 6 \pm 1.1$ . Subsequently, Merritt et al. (2005) showed that Einasto's  $r^{1/n}$  model performed as well as the (1, 3,  $\gamma$ ) model, and gave better fits for the dwarf- and galaxy-sized halos, obtaining  $n \approx 5.6 \pm 0.7$ . For a sample of galaxy-sized halos, Prada et al. (2005) obtained similar values of  $6 - 7.5$ .

Figure 12 shows the application of equation (20) to the  $N$ -body halos of Section 3. A comparison with the (1, 3,  $\gamma$ ) model fits in Figure 5 reveals that Einasto's model provides a better description for five of the six cluster-sized halos, three of the four galaxy-sized halos, and both of the spherical collapse halos.

<sup>3</sup>A similar expression is given in Mamon & Lokas 2005, their equation (A2); and Cardone et al. 2005, their equation (11).

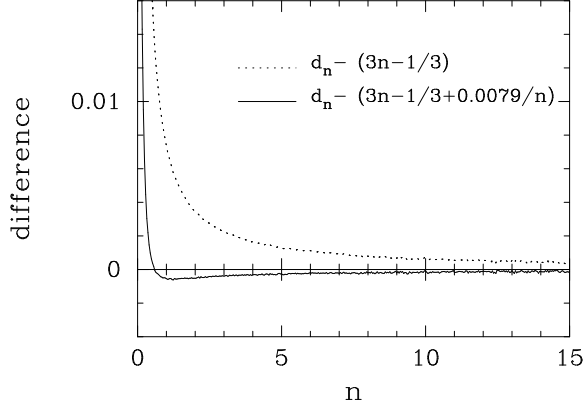


Fig. 11.— Difference between the exact value for  $d_n$  from equation (20), such that  $\Gamma(3n) = 2\gamma(3n, d_n)$ , and the two approximations inset in the Figure.

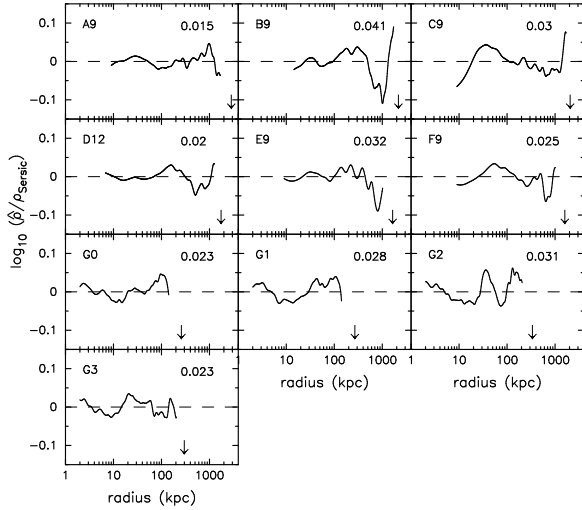


Fig. 12.— Residual profiles from application of Einasto’s  $r^{1/n}$  model (equation 20) to our dark matter density profiles.

Navarro et al. (2004) wrote “adjusting the parameter  $[n]$  allows the profile to be tailored to each individual halo, resulting in improved fits”<sup>4</sup>. Such a breaking of structural homology (see Graham & Colless 1997 for an analogy with projected luminosity profiles) replaces the notion that a universal density profile may exist.

A number of useful expressions pertaining to Einasto’s model, when used as a density profile (equation 20), are given in Cardone et al. (2005) and Mamon & Lokas (2005). In particular, Cardone et al. provide the gravitational potential, as well as approximations to the surface density and space velocity dispersion of the Einasto  $r^{1/n}$  model, while Mamon & Lokas give approximations for the concentration parameter, central density, and  $M_{\text{virial}}/M_{\text{total}}$ . The nature of the inner profile slope of Einasto’s  $r^{1/n}$  model and several other useful quantities are presented in Paper II.

#### 4.3. Prugniel-Simien model: A deprojected Sérsic $R^{1/n}$ model

Merritt et al. (2005) tested how well a deprojected Sérsic  $R^{1/n}$  model fit  $\rho(r)$  from the Navarro et al. (2004)  $N$ -body halos. This was essentially the same as comparing the halo *surface* densities with Sérsic’s  $R^{1/n}$  law. Prugniel & Simien (1997) presented a simple, analytical approximation to the deprojected Sérsic law (their eq. B6):

$$\rho(r) = \rho' \left( \frac{r}{R_e} \right)^{-p} \exp \left[ -b (r/R_e)^{1/n} \right], \quad (25)$$

with

$$\rho' = \frac{M}{L} I_e e^b b^{n(1-p)} \frac{\Gamma(2n)}{2R_e \Gamma(n(3-p))}. \quad (26)$$

Equation (25) is a generalization of equation (2) in Mellier & Mathez (1987), who considered only approximations to the deprojected  $R^{1/4}$  law. Mellier & Mathez’s model was itself a modification of equation (33) from Young (1976), which derived from the work of Poveda, Iturriaga, & Orozco (1960).

In these expressions,  $R_e$ ,  $n$  and  $b$  are understood to be essentially the same quantities that appear

<sup>4</sup>The value of  $n$ , equal to  $1/\alpha$  in Navarro et al.’s (2004) notation, ranged from 4.6 to 8.2 (Navarro et al. 2004, their table 3).



in the Sérsic  $R^{1/n}$  law that describes the projected density (equation 18). In fact, since equation (25) is not exactly a deprojected Sérsic profile, the correspondence between the parameters will not be perfect. We follow the practice of earlier authors and define  $b$  to have the same relation to  $n$  as in equation (19). (For clarity, we have dropped the subscript  $n$  from  $b_n$ .) Although the parameter  $\rho'$  is obtained from fitting the density profile, it can be defined in such a way that the total (finite) mass from equation (25) equals that from equation (18), giving equation (26). (We stress that the  $n$  in the Prugniel-Simien profile is *not* equivalent to the  $n$  in equation (20), Einasto's model.)

This leaves the parameter  $p$ . We define  $p$ , like  $b$ , uniquely in terms of  $n$ :

$$p = 1.0 - 0.6097/n + 0.05463/n^2. \quad (27)$$

Lima Neto et al. (1999) derived this expression by requiring the projection of equation (25) to approximate as closely as possible to the Sérsic profile with the same  $(R_e, n)$ , for  $0.6 \leq n \leq 10$  and  $10^{-2} \leq R/R_e \leq 10^3$ .<sup>5</sup> The accuracy of Prugniel & Simien's (1997) approximation, using equation (27) for  $p(n)$ , is shown in Figure 13.

Terzić & Graham (2005) give simple expressions, in terms of elementary and special functions, for the gravitational potential and force of a galaxy obeying the Prugniel-Simien law, and derive the spatial and line-of-sight velocity dispersion profiles.

One could also allow  $p$  to be a free parameter, creating a density profile that has any desired inner slope. For instance, setting  $p = 0$ , the Prugniel-Simien model reduces to the Einasto model. We do not explore that idea further here.

The density at  $r = R_e$  is given by  $\rho_e = \rho' e^{-b}$ , while the projected surface density at  $R = R_e$ , denoted by  $I_e$ , can be solved for using equation (26). Thus, one can immediately construct (a good approximation to) the projected mass distribution, which will have a Sérsic form (equation 18) with parameters  $(R_e, I_e, \text{ and } n)$ . This allows the halo parameters to be directly compared with those of Sérsic fits to luminous galaxies, which we do in Paper III. In Paper II we recast this model using the

radius where the logarithmic slope of the density profile equals  $-2$ .

The mass profile (Terzić & Graham 2005, their Appendix A; see also Lima Neto et al. 1999 and Márquez et al. 2001), can be written as

$$M(r) = 4\pi\rho'R_e^3 n b^{n(p-3)} \gamma(n(3-p), Z), \quad (28)$$

where  $Z \equiv b(r/R_e)^{1/n}$  and  $\gamma(a, x)$  is the incomplete gamma function given in equation (23). The total mass is obtained by replacing  $\gamma(n(3-p), Z)$  with  $\Gamma(n(3-p))$ , and the circular velocity is given by  $v_{\text{circ}}(r) = \sqrt{GM(r)/r}$ .

In Figure 14, equation (25) has been applied to our dark matter profiles. The average ( $\pm$  standard deviation) of the shape parameter for the galaxy-sized and cluster-sized halos is  $n = 3.59(\pm 0.65)$  and  $n = 2.89(\pm 0.49)$ , respectively. Merritt et al. (2005, their Table 1) found values of  $3.40 \pm 0.36$  and  $2.99 \pm 0.49$  for their sample of galaxies and clusters, respectively, in good agreement with the results obtained here using a different set of  $N$ -body simulations and equation (25), rather than a numerically deprojected  $R^{1/n}$  light profile.

Figure 14 reveals that CDM halos resemble galaxies (Merritt et al. 2005), since the projection of the Prugniel-Simien model closely matches the Sérsic  $R^{1/n}$  model, and the latter is a good approximation to the luminosity profiles of stellar spheroids. Subject to vertical and horizontal scaling, CDM halos have similar mass distributions to elliptical galaxies with an absolute  $B$ -band magnitude around  $-18 \pm 1$  mag; these galaxies have  $n \sim 3$  (see Graham & Guzmán 2003, their figure 9). This result was obscured until recently due to the use of different empirical models by observers and modelers.

Before moving on, we again remark that we have not explored potential refinements to the expression (27) for the quantity  $p$ , but note that this could result in a better matching of the model to the simulated profiles at small radii. As the resolution of  $N$ -body clustering simulations continues to improve, it will make sense to explore such generalizations.

## 5. Model comparison: Which did best?

Table 4 summarizes how well each parametric model performed by listing, for each type of halo, the rms value of  $\Delta$  (equation 12a) for each set of

<sup>5</sup>The value of  $p$  given in equation (27) is preferable to the value  $1.0 - 0.6097/n + 0.05563/n^2$  given in Márquez et al. (2000), (Lima Neto 2005, priv.comm.).

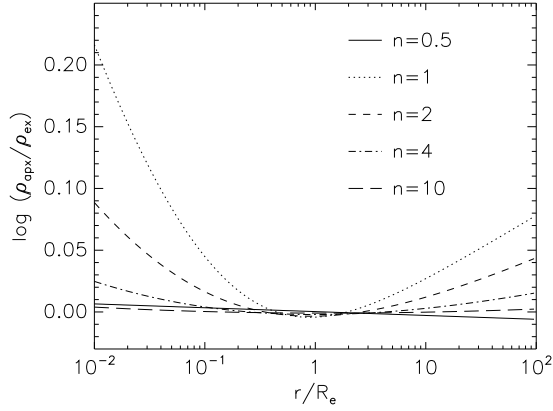


Fig. 13.— Logarithmic difference between the exact deprojection of Sérsic’s  $R^{1/n}$  model (equation 18) and the approximation given by Prugniel & Simien (1997) in equation (25), using the values of  $p$  and  $b$  given in equations (27) and (19), respectively.

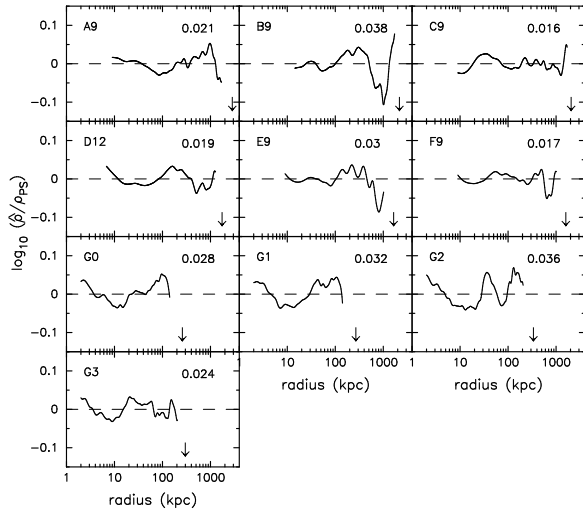


Fig. 14.— Residual profiles from application of the Prugniel & Simien model (equation 25) to our dark matter density profiles.

halos, given by

$$\Delta_{\text{rms}} = \sqrt{\frac{1}{N} \sum_{i=1}^N \Delta_i^2}, \quad (29)$$

with  $N = 6, 4$  and  $2$  for the cluster-sized, galaxy-sized, and spherical-collapse halos, respectively. A detailed description of each model’s performance follows.

The bowl- and hump-shaped residual profiles associated with the 2-parameter model of Burkert (1995) reveal this model’s inability to describe the radial mass distribution in our simulated dark matter halos. The 2-parameter model of Dehnen & McLaughlin (2005) performs considerably better, although it too fails to describe the cold collapse systems and two of the six cluster-sized halos, specifically C09 and F09. Although this (4/9, 31/9, 7/9) model never provides the best fit, it does equal or out-perform the NFW-like  $(1, 3, \gamma)$  model in describing 3 of the 12 halos (A09, D12, G03).

In general, all of the 3-parameter models perform well ( $0.015 \lesssim \Delta \lesssim 0.04$  dex) at fitting the  $N$ -body (non-collapse) halos. However, neither the  $(1, 3, \gamma)$  model nor the 3-parameter Dehnen-McLaughlin model can match the curvature in the density profiles of the cold collapse systems (M11 & M35). On the other hand, both Einasto’s  $r^{1/n}$  model and that from Prugniel & Simien give reasonably good fits ( $\Delta \sim 0.05$  dex) for these two halos.

The Prugniel-Simien model provided the best overall description of the cluster-sized,  $N$ -body halos. The  $(1, 3, \gamma)$  model and the 3-parameter Dehnen-McLaughlin model provided the best fit for only one cluster-sized,  $N$ -body halo each, and even then the  $(1, 3, \gamma)$  model only just out-performed the Prugniel-Simien model which gave the best fit for four of the six cluster-sized halos. For two of these halos, the size of the residual about the optimal Prugniel-Simien fit was roughly half of the value obtained when using the  $(1, 3, \gamma)$  model.

The implication of this result is that Sérsic’s  $R^{1/n}$  model will describe the projected surface density of the cluster-sized, dark matter halos. Intriguingly, Demarco et al. (2003) and Durret, Lima Neto & Forman (2005) have observed that the (projected) hot X-ray gas distribution in clus-

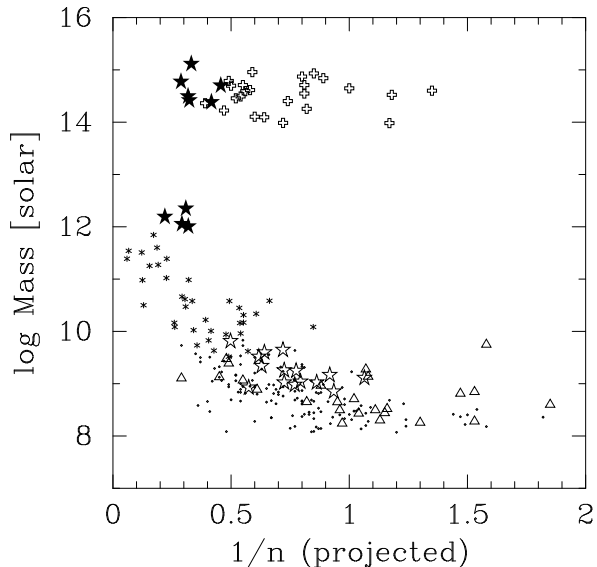


Fig. 15.— Mass versus profile shape ( $1/n$ ). For the galaxies and galaxy clusters, the shape parameters  $n$  have come from the best-fitting Sérsic  $R^{1/n}$  model to the (projected) luminosity- and X-ray profiles, respectively. The galaxy stellar masses, and cluster gas masses are shown here. For DM halos, the virial masses are shown and the shape parameters have come from the best-fitting Prugniel-Simien model. (Note: The value of  $1/n$  from the Prugniel-Simien model applied to a density profile is equivalent to the value of  $n$  from Sérsic’s model applied to the projected distribution.) We are plotting baryonic properties for the galaxies alongside dark matter properties for the simulated halos. Filled stars:  $N$ -body, dark matter halos from this paper; open plus signs: galaxy clusters from Demarco et al. (2003); dots: dwarf Elliptical (dE) galaxies from Binggeli & Jerjen (1998); triangles: dE galaxies from Stiavelli et al. (2001); open stars: dE galaxies from Graham & Guzmán (2003); asterisk: intermediate to bright elliptical galaxies from Caon et al. (1993) and D’Onofrio et al. (1994).

ters can indeed be described with Sérsic’s  $R^{1/n}$  model; although the gas can at times display a rather unrelaxed behavior (Statler & Diehl 2006). Studies of gravitational lensing may therefore benefit from the use of Sérsic’s  $R^{1/n}$  model for which the lensing equation has been solved (Cardone et al. 2004) and for which numerous other properties have previously been computed (Graham & Driver 2005).

With regard to the galaxy-sized,  $N$ -body halos, the situation is somewhat different. Dehnen & McLaughlin’s (2005) anisotropic 3-parameter model provided the best fit for three of the four profiles, with the Einasto  $r^{1/n}$  model providing the best fit for the fourth profile. We also observe that Einasto’s model provided better fits than the  $(1, 3, \gamma)$  model for three of the four  $N$ -body halos. If this observation holds, namely, that the Prugniel-Simien model describes the density profiles of the cluster-sized halos best, while Dehnen & McLaughlin’s 3-parameter model provides the best description of the galaxy-sized halos, it would imply that these halos do not have the same structural form. Of course, even if the same model *did* provide the best fit for both types of halo, any variation in the value of the profile shape  $n$ , or central isotropy parameter  $\beta_0$ , would point toward the existence of nonhomology.

While halos of different mass may be systematically better described by different density laws, it is important to emphasize that a *single* density law provides a good fit to *all* of the  $N$ -body halos considered here. As Table 4 shows, Einasto’s  $r^{1/n}$  law has the smallest, or second-smallest, value of  $\Delta_{rms}$  for galaxy-sized, cluster-sized, and spherical-collapse halos. None of the other parametric models which we considered performs as well “across the board.” The next best performer overall is the Prugniel-Simien profile.

## 6. Discussion

Figure 15 shows our  $N$ -body halos, together with real elliptical galaxies and clusters, in the profile shape vs. mass plane. The profile shape parameter plotted there is either  $n$  from the Sérsic  $R^{1/n}$  model fit to the light profile, or the corresponding parameter from the Prugniel-Simien model fit to the dark-matter density. Dynamical masses from the Demarco et al. (2003) study

of galaxy clusters are shown. We have also included the elliptical galaxy compilation in Graham & Guzmán (2003), converting their  $B$ -band luminosities into solar masses using a stellar mass-to-light ratio of 5.3 (Worthey 1994, for a 12 Gyr old SSP), and an absolute  $B$ -band magnitude for the Sun of 5.47  $B$ -mag (Cox 2000). This approach ignores the contribution from dark matter in galaxies. However, given the uncertainties on how  $M_{\text{tot}}/L$  varies with  $L$  (e.g., Trujillo, Burkert, & Bell 2004, and references therein) we prefer not to apply this correction, and note that the galaxy masses in Figure 15 only reflect the stellar mass.

Figure 15 suggests that the simulated galaxy-sized halos have a different shape parameter, i.e. a different mass distribution, than the simulated cluster-sized halos. The same conclusion was reached by Merritt et al. (2005) who studied a different sample of  $N$ -body halos. The sample of dwarf- and galaxy-sized halos from that paper had a mean ( $\pm$  standard deviation)<sup>6</sup> profile shape  $n = 3.04(\pm 0.34)$ , while the cluster-sized halos had  $n = 2.38(\pm 0.25)$ . We observe this same systematic difference in our  $N$ -body halos. Taking the profile shape  $n$  from the Prugniel-Simien model fits to the density profile (equivalent to the value of  $n$  obtained by fitting Sérsic’s  $R^{1/n}$  model to the projected distribution) we find  $n = 3.59(\pm 0.65)$  for our cluster-sized halos and  $n = 2.89(\pm 0.49)$  for our galaxy-sized halos. A Student  $t$  test, without assuming equal variance in the two distributions, reveals the above means are different at the 88% level. Applying the same test to the data of Merritt et al. (2005; their Table 1, column 2), which is double the size of our sample and also contains dwarf galaxy-sized halos, we find that the means are different at the 99.98% level. We conclude that there is a significant mass dependence in the density profiles of simulated dark-matter halos. Density profiles of more massive halos exhibit more curvature (smaller  $n$ ) on a log-log plot.

The fact that  $n$  varies systematically with halo mass raises the question of which density scale and radial scale to use when characterizing halo structure. In the presence of a “universal” density profile, the ratio between  $R_e$  and  $r_{-2}$  (the radius where the logarithmic slope of the density

profile equals  $-2$ , see Paper II) is a constant factor, but with varying values of  $n$  this is not the case. This remark also holds for the scale density, which is used to measure the contrast with the background density of the universe and provides the so-called “halo concentration.” This in turn raises the question of what “concentration” should actually be used, and whether systematic biases exist if one uses  $\rho_{-2}$  rather than, say,  $\rho_e$ . To reiterate this point: the density ratio between  $r = r_{-2}$  and  $r = R_e$  depends on the profile shape  $n$ , and thus, apparently, on the halo mass.

In Figure 16 we show how the use of  $r_{-2}$  and  $R_e$  produce slightly different results in the size-density diagram (e.g., Figure 8 of Navarro et al. 2004). The relation between size (or equivalently mass) and central concentration (or density) varies depending on how one chooses to measure the sizes of the halos.

To better explore how the homology (i.e., universality) of CDM halos is broken, it would be beneficial to analyze a large, low-resolution sample of halos from a cosmological cube simulation in order to obtain good statistics. Moreover, the collective impact from differing degrees of virialization in the outer regions, possible debris wakes from larger structures, global ringing induced by the last major merger, triaxiality, and the presence of large subhalos could be quantified.

## 7. Summary

We presented a nonparametric algorithm for extracting smooth and continuous representations of spherical density profiles from  $N$ -body data, and applied it to a sample of simulated, dark matter halos. All halos exhibit a continuous variation of logarithmic density slope with radius; in the case of the  $\Lambda$ CDM halos, the variation of slope with radius is close to a power law. We then compared the ability of a variety of parametric models to reproduce the nonparametric  $\rho(r)$ ’s. Over the fitted radial range  $0.01 \lesssim r/r_{\text{vir}} < 1$ , both the Einasto  $r^{1/n}$  model (identical in functional form to Sérsic’s model but expressed in terms of space, rather than projected, radius and density) and the Prugniel-Simien model (an analytical approximation to a de-projected Sérsic law) provide a better description of the data than the  $(1, 3, \gamma)$  model, i.e. the NFW-like double power-law model with in-

<sup>6</sup>Reminder: the uncertainty on the mean is not equal to the standard deviation.

TABLE 4  
RESIDUAL SCATTER: RMS VALUES OF  $\Delta$ .

Model	Cluster-sized halos	Galaxy-sized halos	Spherical-collapse halos
3-parameter models			
Einasto	<b>0.028</b>	<b>0.026</b>	<b>0.052</b>
Prugniel-Simien	<b>0.025</b>	0.030	<b>0.056</b>
(1, 3, $\gamma$ )	0.032	0.028	0.236
Dehnen-McLaughlin (Eq.17)	0.034	<b>0.023</b>	0.193
2-parameter models			
Dehnen-McLaughlin (Eq.16)	0.053	0.032	0.257
NFW	0.046	0.046	0.246
Burkert	0.218	0.164	0.217

NOTE.—Col.(1): Model. Col.(2): rms of the 6 residual scatters,  $\Delta_{\text{rms}}$  (equation 29), for the cluster-sized halos. Col.(3): Similar to Col.(2) but for the 4 galaxy-sized halos. Col.(4): Similar to Col.(2) but for the 2 spherical collapse halos. For each halo type, the two models which perform the best are highlighted in bold.

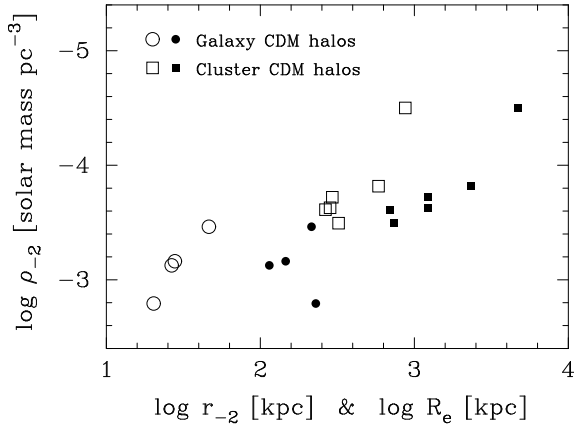


Fig. 16.— The density,  $\rho_{-2}$ , where the logarithmic slope of the density profile equals  $-2$  is plotted against i) the radius where this occurs (open symbols), and ii) the effective radius (filled symbols) derived from the best-fitting Prugniel-Simien model (equation 25). Both  $\rho_{-2}$  and  $r_{-2}$  are also computed from the best-fitting Prugniel-Simien model, see Paper II. If a universal profile existed for these halos, then the vertical difference should be constant for all halos.

ner slope  $\gamma$ . Moreover, unlike the  $(1, 3, \gamma)$  model, both of these models have finite total mass, and are also capable of describing the density profiles of halos formed from the cold collapse of a spherical over-density (Figure 6).

The single function that provides the best overall fit to the halo density profiles is Einasto’s law, equation (20):

$$\rho(r) = \rho_e \exp \left\{ -d_n \left[ (r/r_e)^{1/n} - 1 \right] \right\}$$

with  $d_n$  defined as in equation (24). This conclusion is consistent with that of an earlier study (Merritt et al. 2005) that was based on a different set of  $N$ -body halos. Typical values of the “shape” parameter  $n$  in equation (20) are  $4 \lesssim n \lesssim 7$  (Table 1). Corresponding  $n$  values from Sérsic profile fits to the projected (surface) density range from  $\sim 3$  to  $\sim 3.5$  (Fig. 15).

We propose that Einasto’s model, equation (20), be more widely used to characterize the density profiles of  $N$ -body halos. As noted above, Einasto’s model has already found application in a number of observationally-motivated studies of the distribution of mass in galaxies and galaxy clusters. We propose also that the suitability of Einasto’s model for describing the luminous density profiles of galaxies should be evaluated – either by projecting equation (20) onto the plane of the sky, or by comparing equation (20) directly

with deprojected luminosity profiles. Such a study could strengthen the already strong connection between the density profiles of galaxies and  $N$ -body dark-matter halos (Merritt et al. 2005).

While equation (20) is a good description of all of the halo models considered here, we found that systematic differences do exist in the best-fit models that describe  $N$ -body halos formed via hierarchical merging on the one hand, and those formed via spherical collapse on the other hand, in the sense that the latter have substantially smaller shape parameters,  $n \approx 3.3$  (Table 1). That is, the density profiles in the cold collapse halos decline more quickly than  $r^{-3}$  at large radii, and have shallower inner profile slopes than those produced in simulations of hierarchical merging.

With regard just to the non-collapse models, we also found systematic differences between the cluster- and galaxy-sized halos. The latter are slightly better fit by the 3-parameter Dehnen-McLaughlin model, and the former are slightly better fit by the Prugniel-Simien model (Table 4). This, together with the observation that more massive halos tend to have smaller shape parameters  $n$  (Figure 15), suggests that there may not be a truly “universal” density profile that describes  $\Lambda$ CDM halos.

We kindly thank Gary Mamon for his detailed comments on this manuscript, as well as a second, anonymous referee. We are additionally grateful to Walter Dehnen and Dean McLaughlin for their helpful corrections and comments, and to Carlo Nipoti and Luca Ciotti. We also wish to thank Peeter Tenjes for tracking down and kindly faxing us copies of Einasto’s original papers in Russian. A.G. acknowledges support from NASA grant HST-AR-09927.01-A from the Space Telescope Science Institute, and the Australian Research Council through Discovery Project Grant DP0451426. D.M. was supported by grants AST 04-20920 and AST 04-37519 from the National Science Foundation and grant NNG04GJ48G from NASA. J.D. is grateful for financial support from the Swiss National Science Foundation. B.T. acknowledges support from Department of Energy grant G1A62056.

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